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# Spin $3/2$ pentaquarks

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Abstract: We investigate the possible existence of the spin 3/2 pentaquark states using interpolating currents with K-N color-octet structure in the framework of QCD finite energy sum rule (FESR). We pay special attention to the convergence of the operator product expansion.

Keywords: Phenomenological Models, QCD, Sum Rules.

## Contents





## 1. Introduction

Since LEPS collaboration reported the $\Theta^+$ pentaquark [1], many subsequent experiments claimed the confirmation of this exotic baryon with S=+1 while many other groups didn't observe it in their search. Experiments with both positive and negative results are reviewed in refs. [2, 3]. At present, neither the existence nor the non-existence of $\Theta^+$ is established, which can only be settled by the new high-statistical experiments.

If $\Theta^+$ *really* exists, its low-lying mass, extremely narrow width and weird production mechanism pose a serious challenge to theorists. There have been over four hundred theoretical papers addressing these issues. The early theoretical development can be found in a recent review [4]. The angular moment of $\Theta^+$ is assumed to be $J = \frac{1}{2}$ in order to render $\Theta^+$ low-lying in most of these efforts. Moreover, some models prefer the positive parity to ensure a narrow $\Theta^+$.

However, the possibility of $J = \frac{3}{2}$ is not excluded. For example, $J^P = \frac{1}{2}^+$ pentaquarks are always accompanied by $J^P = \frac{3}{2}^+$ partners [6] in Jaffe and Wilcezk's diquark model [5]. Their magnetic moments were calculated in [7] while their radiative decays and photoproduction were studied in [8].

If $\Theta^+$ carries $J^P = \frac{3}{2}^-$, it decays into nucleons and kaons via D-wave. The phase space suppression factor is $\sim (p_K/m_\Theta)^5 \sim 10^{-5}$ where $p_K$ is the decay momentum of the kaon. The decay width could be well below ten MeV even if the coupling constant $G_{\Theta KN}$ is big due to the absence of the orbital excitation inside $\Theta^+$ and $\Theta$'s strong overlap with $KN$.

There have been a few theoretical papers on the possible $J = \frac{3}{2}$ pentaquarks using different models. Page and Robert suggested $I = 2, J^P = \frac{1}{2}^-, \frac{3}{2}^-$ for $\Theta^+$ to resolve the narrow width puzzle [9]. Jaffe and Wilczek discussed the $J^P = \frac{3}{2}^-$ assignment for $\Xi$

pentaquark [10]. The mass spectrum of $J^P = \frac{3}{2}^-$ pentaquarks were studied with the perturbative chiral quark model [11]. Takeuchi and Shimizu suggested the observed $\Theta$ resonance as a $I = 0, J^P = \frac{3}{2}^-$ $NK^*$ bound state using the quark model [12]. With the flux tube model, Kanada-Enyo et al. studied the mass and decay width of the $I = 1, J^P = \frac{3}{2}^-$ pentaquark [13]. Huang et al. proposed $\Theta^+$ as a molecular state of $NK\pi$ with $I = 1, J^P = \frac{3}{2}^-$ using the chiral SU(3) quark model [14]. The phenomenology of $J = \frac{3}{2}$ pentaquarks such as the mixing scheme and mass pattern was discussed in [15]. Pentaquark states with J=3/2 and I=0,1 were studied using currents composed of one scalar diquark and one vector diquark [16].

We shall employ QCD sum rules (QSR) to explore the possible existence of the $J = \frac{3}{2}, S = +1$ pentaquark states with the isospin $I = 0, 1, 2$. QSR formalism was first employed to study the pentaquark mass with different isospin in ref. [17]. Up to now there have been more than ten papers on pentaquarks within this framework [18–26, 16, 27–29].

In practice there are two forms of QCD sum rules. The first one is the conventional Laplace sum rules introduced originally by the inventors of this formalism [30]. The other one is the finite energy sum rule (FESR) [31]. Their difference lies in the weight function. The right-hand-side (RHS) of the traditional sum rules deals with $\int_{s_{min}}^{s_0} \rho(s) e^{-s/M_B^2} ds$, where $\rho(s)$ is the spectral density including the nonperturbative power corrections arising from various condensates. $M_B$ is the Borel parameter. $s_{min}$ is the starting point of the integral, which is zero for massless quarks. In the analysis of the sum rules, the quark-hadron duality is always invoked. Starting from $s_0$, the physical spectral density, which arises from the higher resonances and continuum is always replaced by the perturbative one. Hence $s_0$ is called the threshold parameter and is typically around the radial excitation mass.

For the FESR approach, the exponential weight function $e^{-s/M_B^2}$ is replaced by $s^n$ in the numerical analysis. For the conventional ground-state hadrons such as the rho and nucleon, both the Laplace QSR and FESR yield almost the same numerical results for the hadron mass, thanks to (1) the good convergence of the operator product expansion; and (2) the useful experimental guidance on the threshold parameter $s_0$. The reason is simple: the rough value of the radial excitation is more or less known experimentally.

In the present case, the situation is more tricky. Even the existence of the lowest $\Theta^+$ pentaquark has not been established, let alone its radial excitation. On the other hand, the spectral density $\rho(s) \sim s^m$ with $m \geq 5$, which causes strong dependence on the continuum or the threshold parameter $s_0$. Hence, compared to the Laplace sum rule with the double parameters $(s_0, M_B)$, the single-parameter FESR may have some advantage as recently noted in [27]. With FESR, one can study the dependence of $m_\Theta$ on $s_0$ in the working region.

On the other hand, the disadvantage of FESR is that its weight function $s^n$ enhances the continuum part even more than the weight function $e^{-s/M_B^2}$ in the Laplace sum rule. Hence some uncertainty is connected to the continuum threshold $s_0$. Especially one must



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make sure that only the lowest pole contributes to the FESR below $s_0$. Otherwise the result will be misleading, which will be shown explicitly in our numerical analysis. To be more specific, a *naive* stability region in $s_0$ is no guarantee of a *physically reasonable* value for $s_0$. For example, the FESR with an extracted threshold $s_0 \approx 20\,\mathrm{GeV}^2$ is certainly irrelevant for the $\Theta^+$ pentaquark around $1.53\,\mathrm{GeV}$.

If a narrow resonance *really* exists, there should exist (1) a spectral density $\rho(s)$ with reasonably good behavior and (2) some values of $s_0$, on which the dependence of $m_\Theta$ is weak. For example, an oscillating $\rho(s)$ from negative to positive values around $m_\Theta^2$ is regarded as having "bad" behavior. The physical spectral density should take the Breit-Wigner form around $m_\Theta^2$ if $\Theta^+$ *really* exists as a very narrow resonance. Hence, $\rho(s)$ should be either fully positive-definite or negative-definite around $m_\Theta^2$.

We will use FESR to analyze the possible existence of $J=\frac{3}{2}, S=+1$ pentaquarks in this work. This paper is organized as follows. In section 2, we construct the interpolating currents with different isospin and present the formalism. The spectral densities and numerical analysis are given in section 3. The last section is a short discussion.

## 2. Formalism of FESR

The method of QCD sum rules [30, 32, 33] incorporates two basic properties of QCD in the low energy domain: confinement and approximate chiral symmetry and its spontaneous breaking. One considers a correlation function of some specific interpolating currents with the proper quantum numbers and calculates the correlator perturbatively starting from high energy region. Then the resonance region is approached where non-perturbative corrections in terms of various condensates gradually become important. Using the operator product expansion, the spectral density of the correlator at the quark gluon level can be obtained in QCD. On the other hand, the spectral density can be expressed in term of physical observables like masses, decay constants, coupling constants etc at the hadron level. With the assumption of quark hadron duality these two spectral densities can be related to each other. In this way one can extract hadron masses etc.

We use the following interpolating current for the $I=0, S=+1, J=\frac{3}{2}$ pentaquark state

$$\eta^0_{\mu,K-N}(x) = \frac{1}{\sqrt{2}}\varepsilon^{abc}\Big[u_a^T(x)C\gamma_5 d_b(x)\Big]\Big\{u_e(x)\bar{s}_e(x)\gamma_\mu d_c(x) - d_e(x)\bar{s}_e(x)\gamma_\mu u_c(x)\Big\} \tag{2.1}$$

where three quarks and the remaining $\bar{q}q$ pair are both in a color adjoint representation [17]. Similarly, we can introduce the $I=1, J=\frac{3}{2}$ current

$$\eta^1_{\mu,K-N}(x) = \frac{1}{\sqrt{2}}\varepsilon^{abc}\Big[u_a^T(x)C\gamma_5 d_b(x)\Big]\Big\{u_e(x)\bar{s}_e(x)\gamma_\mu d_c(x) + d_e(x)\bar{s}_e(x)\gamma_\mu u_c(x)\Big\}\,. \tag{2.2}$$

For $I=2, I_z=2, J=\frac{3}{2}$ state, we use

$$\eta^2_{\mu,K-N}(x) = \Big[u_a^T(x)C\gamma_\nu u_b(x)\Big]\gamma^\nu\gamma_5 u_e(x)\bar{s}_e(x)\gamma_\mu u_c(x)\,. \tag{2.3}$$

The overlapping amplitude $f_j$ of the interpolating current is defined as

$$\langle 0|\eta_\mu(0)|\frac{3}{2}, p, j\rangle = f_j \upsilon_\mu(p)\,, \tag{2.4}$$

where $j$ is the isospin. $\upsilon_\mu$ is the Rarita-Schwinger spinor for the $J = \frac{3}{2}$ pentaquark, which satisfies $(\hat{p} - M_X)\upsilon_\mu = 0$, $\bar{\upsilon}_\mu \upsilon^\mu = -2M_X$, and $\gamma_\mu \upsilon^\mu = p_\mu \upsilon^\mu = 0$.

We consider the following correlation function

$$i \int d^4x e^{-ipx} \langle 0|T\{\eta_\mu(x)\bar{\eta_\nu}(0)\}|0\rangle = g_{\mu\nu}\left(\Pi_A(p^2)\hat{p} + \Pi_B(p^2)\right) + \cdots \tag{2.5}$$

where the ellipse denotes other Lorentz structures which receive contributions from both $J = \frac{1}{2}$ and $J = \frac{3}{2}$ resonances. The tensor structures $g_{\mu\nu}, g_{\mu\nu}\hat{p}$ are particular. They receive contribution only from the $J = \frac{3}{2}$ pentaquarks.

We can write a dispersion relation for the scalar functions $\Pi^{A,B}(p^2)$.

$$\Pi_{A,B}(p^2) = \int ds \frac{\rho_{A,B}(s)}{(s - p^2 - i\epsilon)}\,, \tag{2.6}$$

where $\rho_{A,B}(s)$ is the spectral density.

The width of the $\Theta^+$ pentaquark is less than several MeV. For such a narrow resonance, its spectral density can be approximated by a delta function very well. Hence, at the hadron level we have

$$\begin{aligned} \rho_A(s) &= f_A^2 \delta(s - M_j^2) + \text{higher states} \\ \rho_B(s) &= f_B^2 M_j \delta(s - M_j^2) + \text{higher states}\,, \end{aligned} \tag{2.7}$$

where $M_j$ is the pentaquark mass. In principle, there also exists non-resonant kaon nucleon continuum contribution to $\rho_{A,B}(s)$ since $\Theta^+$ lies above threshold. However, this kind of non-resonant K N continuum is either of D-wave for $J^P = \frac{3}{2}^-$ or of P-wave for $J^P = \frac{3}{2}^+$. Their contribution is strongly suppressed compared to the resonant $\Theta^+$ pole contribution. Here we want to emphasize that

$$f_B^2 = +f_A^2 \tag{2.8}$$

for $J^P = \frac{3}{2}^+$ pentaquarks while

$$f_B^2 = -f_A^2 \tag{2.9}$$

for $J^P = \frac{3}{2}^-$ pentaquarks. Hence the relative sign between $f_B^2$ and $|f_A|^2$ indicates the parity of the corresponding pentaquark.

On the other hand, the spectral density $\rho_{A,B}(s)$ can be calculated at the quark gluon level. For example, the correlation function for the interpolating current (2.1) reads

$$\begin{aligned} \langle 0|T\{\eta^1_\mu(x)\bar{\eta}^1_\nu(0)\}|0\rangle = \varepsilon^{abc}\varepsilon^{a'b'c'}\Big\{ & 2iS_u^{ea'} C\gamma_5 S_d^{Tcb'} \gamma_\mu^T S_s^{Te'e}(-x)\gamma_\nu^T S_d^{Tbc'} C\gamma_5 S_u^{ae'} - \\ & - iS_u^{ea'} C\gamma_5 S_d^{Tbb'} C\gamma_5 S_u^{ac'} \gamma_\nu S_s^{e'e}(-x)\gamma_\mu S_d^{ce'} - \\ & - iS_u^{ec'} \gamma_\nu S_s^{e'e}(-x)\gamma_\mu S_d^{cb'} C\gamma_5 S_u^{Taa'} C\gamma_5 S_d^{be'} + \\ & + i\mathrm{Tr}\Big[S_u^{Taa'} C\gamma_5 S_d^{bb'} C\gamma_5\Big] S_u^{ec'} \gamma_\nu S_s^{e'e}(-x)\gamma_\mu S_d^{ce'} - \end{aligned}$$

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$$- iS_u^{ea'} C\gamma_5 S_d^{Tbb'} C\gamma_5 S_u^{ae'} \,\mathrm{Tr}\left[S_s^{e'e}(-x)\gamma_\mu S_d^{cc'}\gamma_\nu\right] +$$
$$+ i\,\mathrm{Tr}\left[S_u^{Taa'} C\gamma_5 S_d^{bb'} C\gamma_5\right] \mathrm{Tr}\left[S_s^{e'e}(-x)\gamma_\mu S_d^{cc'}\gamma_\nu\right] S_u^{ee'} -$$
$$- i\,\mathrm{Tr}\left[S_u^{Taa'} C\gamma_5 S_d^{bc'}\gamma_\nu S_s^{e'e}(-x)\gamma_\mu S_d^{cb'} C\gamma_5\right] S_u^{ee'} \Big\}\,, \quad (2.10)$$

where $S(x) = -i\langle 0|T\{q(x)\bar{q}(0)\}|0\rangle$ is the full quark propagator in the coordinate space. Throughout our calculation, we assume the up and down quarks are massless. The first few terms of the quark propagator is

$$iS^{ab}(x) = \frac{i\delta^a b}{2\pi^2 x^4}\hat{x} + \frac{i}{32\pi^2}\frac{\lambda^n_{ab}}{2} g_s G^n_{\mu\nu}\frac{1}{x^2}(\sigma^{\mu\nu}\hat{x} + \hat{x}\sigma^{\mu\nu}) - \frac{\delta^{ab}}{12}\langle\bar{q}q\rangle + \frac{\delta^{ab}x^2}{192}\langle g_s\bar{q}\sigma Gq\rangle + \cdots .$$

First the correlator is calculated in the coordinate space. Then $\Pi_{A,B}(p^2)$ can be derived after making Fourier transformation to $\Pi_{A,B}(x)$. From the imaginary part of $\Pi_{A,B}(p^2)$ one can extract the spectral density $\rho_{A,B}(s)$ at the quark hadron level.

With the spectral density, the $n$th moment of FESR is defined as

$$W_{A,B}(n, s_0) = \int_{m_s^2}^{s_0} ds s^n \rho_{A,B}(s)\,, \quad (2.11)$$

where $n \geq 0$. With the quark hadron duality assumption we get the finite energy sum rule

$$W_{A,B}(n, s_0)|_{Hadron} = W_{A,B}(n, s_0)|_{QCD}\,. \quad (2.12)$$

The mass and $f_j^2$ can be obtained as

$$M_j^2 = \frac{W_{A,B}(n+1, s_0)}{W_{A,B}(n, s_0)} \quad (2.13)$$

$$f_j^2 M_j = W_B(n=0, s_0)|_{QCD}\,. \quad (2.14)$$

In principle, one can extract the threshold from the requirement

$$\frac{dM_j^2}{ds_0} = 0\,, \quad (2.15)$$

or

$$\int_{m_s^2}^{s_0} (s_0 - s) s^n \rho(s) = 0\,. \quad (2.16)$$

If $\rho(s) > 0$ or $\rho(s) < 0$ in the whole region $[m_s^2, \infty)$, there does not exist a stable threshold for this finite energy sum rule .

For the gluon condensates we keep only D=4 term and neglect D=6, 8 pieces in our calculation. The contribution of D=6, 8 gluon condensates was found to be much smaller than D=4 term in previous QSR analysis. For D=7-9 power corrections, we keep only those numerically large terms, which are related to the quark condensate $\langle\bar{q}q\rangle$ or the quark gluon mixed condensate $\langle g_s\bar{q}\sigma Gq\rangle$. Condensates such as $\langle\bar{q}q\rangle\langle g_s^2 GG\rangle$, $\langle g_s\bar{q}\sigma Gq\rangle\langle g_s^2 GG\rangle$ are neglected.

We use the following values of condensates in the numerical analysis: $\langle\bar{q}q\rangle = -(0.24\,\mathrm{GeV})^3$, $\langle\bar{s}s\rangle = -(0.8 \pm 0.1)(0.24\,\mathrm{GeV})^3$, $\langle g_s^2 GG\rangle = (0.48 \pm 0.14)\,\mathrm{GeV}^4$, $\langle g_s\bar{q}\sigma Gq\rangle = -m_0^2 \times \langle\bar{q}q\rangle$, $m_0^2 = (0.8 \pm 0.2)\,\mathrm{GeV}^2$. We use $m_s(1\,\mathrm{GeV}) = 0.15\,\mathrm{GeV}$ for the strange quark mass in the $\overline{MS}$ scheme.

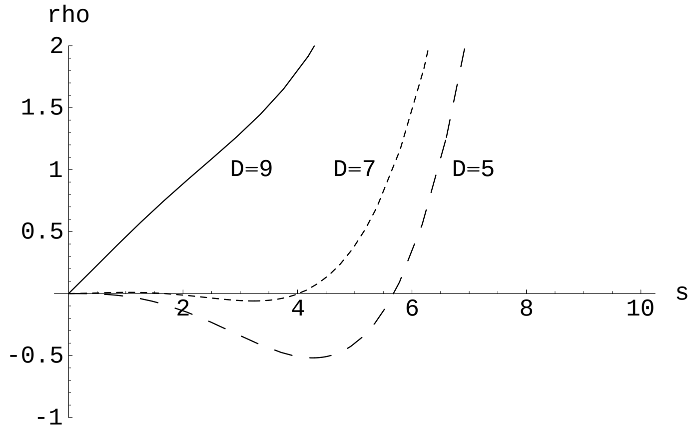


**Figure 1:** The variation of the spectral density $\rho_B^0$ from the tensor structure $g_{\mu\nu}$ (in unit of $\mathrm{GeV}^{11} \times 10^{-8}$) with s (in unit of $\mathrm{GeV}^2$) for the I=0 current (2.1). The long-dashed, short-dashed and solid curves correspond to $\rho_B^0$ with $D = 5, 7, 9$ condensates respectively.

## 3. Numerical analysis

### 3.1 $I = 0$ FESR from $g_{\mu\nu}$ Structure

After tedious calculation, the spectral density $\rho_B^0(s)$ with condensates up to dimension 9 reads

$$\begin{aligned}\rho_B^0(s) = &\frac{1}{2^{19}\cdot 175\cdot \pi^8} s^5 m_s - \frac{5}{2^{16}\cdot 27\cdot \pi^6} s^4 \left(\langle \bar{q}q\rangle + \frac{3}{20}\langle \bar{s}s\rangle\right) + \\ &+ \frac{7}{2^{20}\cdot 45\cdot \pi^8} s^3 m_s \langle g_s^2 GG\rangle - \frac{17}{2^{14}\cdot 45\cdot \pi^6} s^3 \langle \bar{q}\sigma\cdot Gq\rangle + \\ &+ \frac{1}{2^{12}\cdot 3\cdot \pi^4} s^2 m_s \left(\frac{103}{6}\langle \bar{q}q\rangle - 5\langle \bar{s}s\rangle\right)\langle \bar{q}q\rangle - \\ &- \frac{1}{2^8\cdot 27\cdot \pi^2} s(56\langle \bar{q}q\rangle + 75\langle \bar{s}s\rangle)\langle \bar{q}q\rangle^2 ,\end{aligned} \tag{3.1}$$

where we have used the factorization approximation for the high-dimension quark condensates.

Both the perturbative piece and the D=3 power correction from the quark condensate in $\rho_B^0(s)$ are positive. There are two types of D=5 power corrections. One arises from the gluon condensate $m_s\langle g_s^2 GG\rangle$. Its contribution is positive. The other one is from the quark gluon mixed condensate $\langle g_s\bar{q}\sigma Gq\rangle$, which overwhelms the D=3 quark condensate and D=4 gluon condensate in magnitude and carries a minus sign. As can be seen from Fig. 1, the quark gluon mixed condensate renders the spectral density negative for a big range of s. The D=7 condensate yields a positive contribution and cancels the big negative contribution from the quark gluon mixed condensate, leading to a nearly vanishing $\rho_B^0(s)$ for $s \le 4\ \mathrm{GeV}^2$ to this order. The contribution from the D=9 condensate is also large. In fact, $\rho_B^0(s) > 0$ is positive throughout the whole range $[m_s^2, \infty)$ with the inclusion of the D=9 power correction.

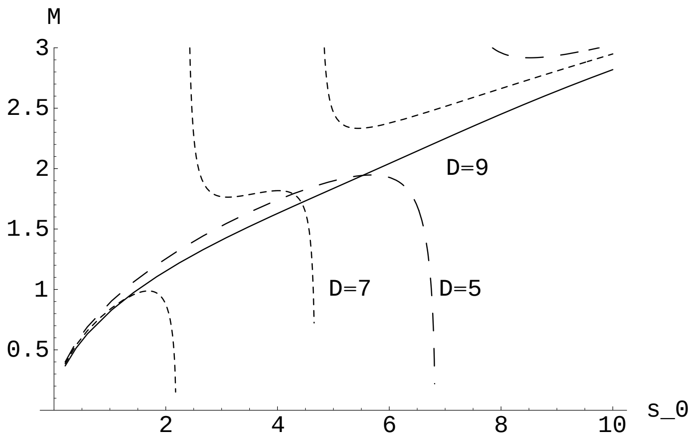


**Figure 2:** The variation of the mass $M$ with the threshold $s_0$ (in unit of $\mathrm{GeV}^2$) corresponding to $\Pi_B$ of the I=0 current (2.1). Same conventions as in figure 1.



It is a common feature in the pentaquark sum rules that the quark gluon mixed condensate plays a very striking, sometimes dominant role when the sum rule is truncated at low orders. In contrast, the ratio between the power corrections from $\langle g_s\bar{q}\sigma Gq\rangle$ and $\langle\bar{q}q\rangle$ is less than $-20\%$ in the nucleon mass sum rule from $\hat{p}$ structure [34], which ensures the convergence of the operator product expansion (OPE).

In order to analyze the OPE convergence in the present case, we list the right hand side (RHS) of eq. (2.12) for the case of $n = 0$ below:

$$W_B^{I=0}(0, s_0) \sim 3.2\times 10^{-3}s_0 + 1 - \frac{7.3}{s_0} + \frac{12}{s_0^2} + \frac{247}{s_0^3}\,, \tag{3.2}$$

where the individual terms are normalized according to the quark condensate.

In the FESR framework the expansion parameter is $1/s_0$ while it's $1/M_B^2$ in the conventional Laplace sum rule analysis. eq. (3.2) indicates that the present FESR is very sensitive to the high dimension condensates with $D \geq 5$. Numerically, these nonperturbative power corrections converge for $s_0 \geq 10\,\mathrm{GeV}^2$ only. Such a big threshold is irrelevant for the $\Theta^+$ pentaquark around 1.53 GeV. We have to conclude that this FESR is not suitable for the extraction of pentaquark mass.

If we ignore the convergence problem, the variation of the pentaquark mass with the continuum threshold is shown in Fig 2. Naively, there exists a "stable" threshold of $s_0 = 5.6\,\mathrm{GeV}^2$ corresponding to $m_\Theta = (1.8\pm0.1)\,\mathrm{GeV}$ if we truncate the sum rule at D=5 order.

However, if we add the D=7 correction, the shape of the curve changes dramatically. With the inclusion of D=9 condensates, there is no stable continuum threshold at all. The underlying reason of the extreme sensitivity of this FESR to the high dimension condensates is the lack of the convergence of the operator product expansion. Hence the extracted pentaquark mass and continuum threshold are very unreliable.

### 3.2 $I = 0$ FESR from $g_{\mu\nu}\hat{p}$ structure

Now let's move to the chirally even tensor structure $g_{\mu\nu}\hat{p}$. $\rho_A^0(s)$ with condensates up to

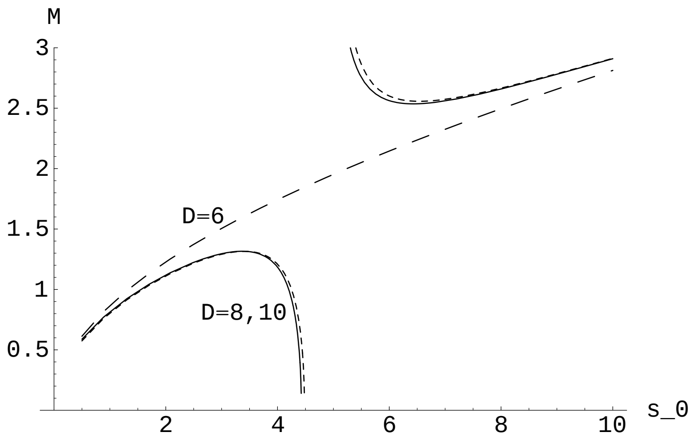


**Figure 3:** The variation of $M$ with $s_0$ corresponding to $\Pi_A$ of the I=0 current (2.1). The long-dashed, short-dashed and solid curves correspond to $\rho_A^0$ with $D = 6, 8, 10$ condensates respectively.

dimension 10 reads

$$\begin{aligned}\rho_A^0(s) = & \frac{17}{2^{17}\cdot 5!\cdot 5!\cdot \pi^8}s^5 - \frac{5}{2^{22}\cdot 81\cdot \pi^8}s^3\langle g_s^2 GG\rangle + \frac{1}{2^{13}\cdot 9\cdot \pi^6}s^3 m_s\left(\frac{17}{48}\langle \bar{s}s\rangle - \frac{7}{5}\langle \bar{q}q\rangle\right) + \\ & + \frac{1}{2^{11}\cdot 9\cdot \pi^4}s^2\left(\frac{121}{20}\langle \bar{q}q\rangle^2 + 7\langle \bar{q}q\rangle\langle \bar{s}s\rangle\right) - \frac{193}{2^{18}\cdot 15\cdot \pi^6}s^2 m_s\langle \bar{q}\sigma\cdot Gq\rangle + \\ & + \frac{1}{2^{14}\cdot 27\cdot \pi^4}s(1021\langle \bar{q}q\rangle + 286\langle \bar{s}s\rangle)\langle \bar{q}\sigma\cdot Gq\rangle + \\ & + \frac{5}{2^6\cdot 9\cdot \pi^2}m_s\left(-\langle \bar{q}q\rangle + \frac{13}{24}\langle \bar{s}s\rangle\right)\langle \bar{q}q\rangle^2 . \end{aligned} \tag{3.3}$$

We divide the zeroth moment from $\rho_A^0(s)$ by the D=6 corrections, which arise mainly from the four quark condensate.

$$W_A^{I=0}(0, s_0) \sim 3.8\times 10^{-4}s_0^3 + 2\times 10^{-2}s_0 + 1 - \frac{5.4}{s_0} + \frac{0.5}{s_0^2} . \tag{3.4}$$

Here the D=8 term, which is the product of $\langle \bar{q}q\rangle$ and $\langle g_s\bar{q}\sigma Gq\rangle$, plays a very important role. The convergence of the OPE requires $s_0 \geq 13\,\mathrm{GeV}^2$, which renders the above FESR useless in the extraction of $m_\Theta$. If we ignore the convergence criteria, we may arrive at rather misleading results of $m_\Theta$ and $s_0$ as shown by the variation of $m_\Theta$ with $s_0$ in Fig. 3 when power corrections with D=6, 8, 10 are included.

### 3.3 I=1 and I=2 Cases

All the above analysis can be extended to $I = 1, 2$ case. Roughly speaking, the same conclusions hold. The spectral densities for the I=1 case are

$$\begin{aligned}\rho_B^1(s) = & \frac{-1}{2^{19}\cdot 175\cdot \pi^8}s^5 m_s - \frac{1}{2^{16}\cdot 27\cdot \pi^6}s^4\left(\langle \bar{q}q\rangle - \frac{3}{4}\langle \bar{s}s\rangle\right) - \\ & - \frac{1}{2^{18}\cdot 15\cdot \pi^8}s^3 m_s\langle g_s^2 GG\rangle - \frac{1}{2^{12}\cdot 45\cdot \pi^6}s^3\langle \bar{q}\sigma\cdot Gq\rangle + \end{aligned}$$

$$+\frac{1}{2^{12}\cdot 3\cdot\pi^4}s^2 m_s\left(\frac{37}{6}\langle\bar{q}q\rangle-\langle\bar{s}s\rangle\right)\langle\bar{q}q\rangle-$$
$$-\frac{1}{2^8\cdot 27\cdot\pi^2}s(24\langle\bar{q}q\rangle+29\langle\bar{s}s\rangle)\langle\bar{q}q\rangle^2 \tag{3.5}$$

and

$$\begin{aligned}\rho^1_A(s) =&\ \frac{11}{2^{17}\cdot 5!\cdot 5!\cdot\pi^8}s^5+\frac{23}{2^{22}\cdot 405\cdot\pi^8}s^3\langle g_s^2GG\rangle+\frac{1}{2^{17}\cdot\pi^6}s^3 m_s\left(\frac{11}{27}\langle\bar{s}s\rangle-\frac{16}{15}\langle\bar{q}q\rangle\right)+\\ &+\frac{1}{2^{11}\cdot 9\cdot\pi^4}s^2\left(\frac{27}{20}\langle\bar{q}q\rangle^2+3\langle\bar{q}q\rangle\langle\bar{s}s\rangle\right)-\frac{89}{2^{18}\cdot 15\cdot\pi^6}s^2 m_s\langle\bar{q}\sigma\cdot Gq\rangle+\\ &+\frac{1}{2^{13}\cdot 27\cdot\pi^4}s(135\langle\bar{q}q\rangle+71\langle\bar{s}s\rangle)\langle\bar{q}\sigma\cdot Gq\rangle+\\ &+\frac{1}{2^6\cdot 9\cdot\pi^2}m_s\left(-\langle\bar{q}q\rangle+\frac{11}{24}\langle\bar{s}s\rangle\right)\langle\bar{q}q\rangle^2 .\end{aligned} \tag{3.6}$$

We also have normalized zeroth moments

$$W_B^{I=1}(0,s_0)\sim -4.4\times 10^{-2}s_0+1-\frac{25}{s_0}+\frac{66}{s_0^2}+\frac{1406}{s_0^3}\,, \tag{3.7}$$
$$W_A^{I=1}(0,s_0)\sim 7.7\times 10^{-4}s_0^3+7.3\times 10^{-3}s_0+1-\frac{5.1}{s_0}+\frac{0.3}{s_0^2}\,. \tag{3.8}$$

For I=2 case, the spectral densities read

$$\begin{aligned}\rho^2_B(s) =&\ \frac{-1}{2^{11}\cdot 27\cdot\pi^6}s^4\langle\bar{q}q\rangle-\frac{1}{2^{15}\cdot 9\cdot\pi^8}s^3 m_s\langle g_s^2GG\rangle\\ &-\frac{1}{2^{10}\cdot 9\cdot\pi^6}s^3\langle\bar{q}\sigma\cdot Gq\rangle+\frac{1}{72\cdot\pi^4}s^2 m_s\left(\langle\bar{q}q\rangle-\frac{3}{16}\langle\bar{s}s\rangle\right)\langle\bar{q}q\rangle-\\ &-\frac{1}{216\cdot\pi^2}s(24\langle\bar{q}q\rangle+23\langle\bar{s}s\rangle)\langle\bar{q}q\rangle^2 ,\end{aligned} \tag{3.9}$$

$$\begin{aligned}\rho^2_A(s) =&\ \frac{7}{2^{12}\cdot 5!\cdot 5!\cdot\pi^8}s^5-\frac{59}{2^{18}\cdot 135\cdot\pi^8}s^3\langle g_s^2GG\rangle+\frac{1}{2^{10}\cdot 3\pi^6}s^3 m_s\left(\frac{7}{36}\langle\bar{s}s\rangle-\frac{4}{5}\langle\bar{q}q\rangle\right)+\\ &+\frac{1}{2^6\cdot 9\pi^4}s^2\left(2\langle\bar{q}q\rangle^2+3\langle\bar{q}q\rangle\langle\bar{s}s\rangle\right)-\frac{499}{2^{13}\cdot 45\cdot\pi^6}s^2 m_s\langle\bar{q}\sigma\cdot Gq\rangle+\\ &+\frac{1}{2^8\cdot 9\cdot\pi^4}s\left(\frac{751}{12}\langle\bar{q}q\rangle+37\langle\bar{s}s\rangle\right)\langle\bar{q}\sigma\cdot Gq\rangle+\\ &+\frac{1}{18\cdot\pi^2}m_s\left(-\langle\bar{q}q\rangle+\frac{11}{12}\langle\bar{s}s\rangle\right)\langle\bar{q}q\rangle^2 .\end{aligned} \tag{3.10}$$

Similarly we have

$$W_B^{I=2}(0,s_0)\sim 1-\frac{6}{s_0}+\frac{22}{s_0^2}+\frac{505}{s_0^3}\,, \tag{3.11}$$
$$W_A^{I=2}(0,s_0)\sim 4.2\times 10^{-4}s_0^3+1.8\times 10^{-2}s_0+1-\frac{6.3}{s_0}+\frac{0.1}{s_0^2}\,. \tag{3.12}$$

For completeness we also present the variation of $m_\Theta$ with $s_0$ in figures 4–7 although they are unreliable due to the lack of convergence of OPE.

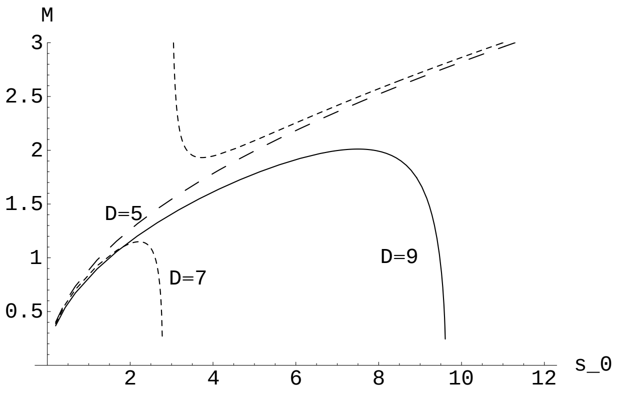


**Figure 4:** The variation of $M$ with $s_0$ corresponding to $\Pi_B$ of the I=1 current (2.2).

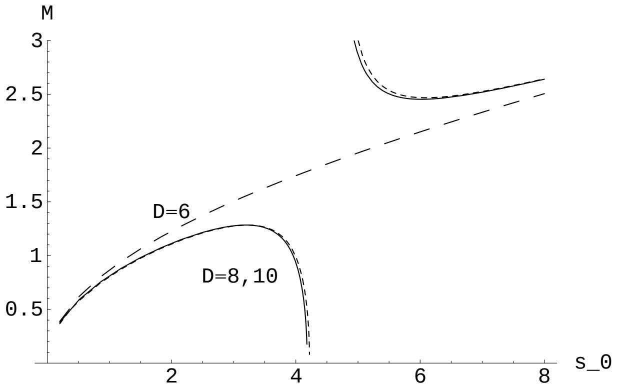


**Figure 5:** The variation of $M$ with $s_0$ corresponding to $\Pi_A$ of the I=1 current (2.2).

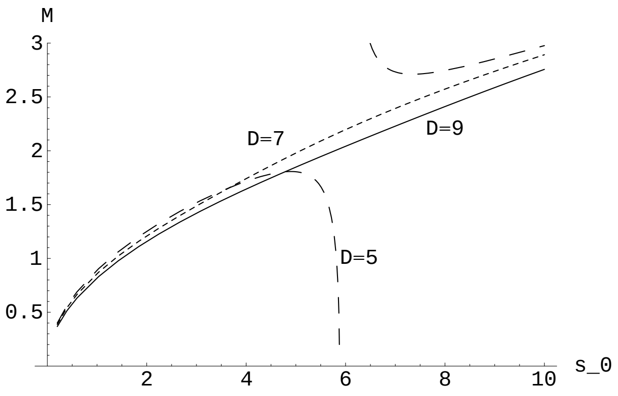


**Figure 6:** The variation of $M$ with $s_0$ corresponding to $\Pi_B$ of the I=2 current (2.3).

## 4. Discussion

In this paper we have constructed the finite energy sum rules for the spin 3/2 pentaquarks using the K-N color-octet type interpolating currents. Both $g_{\mu\nu}$ and $g_{\mu\nu}\hat{p}$ tensor structures are unique for spin 3/2 pentaquarks. Because of the high dimension of the pentaquark

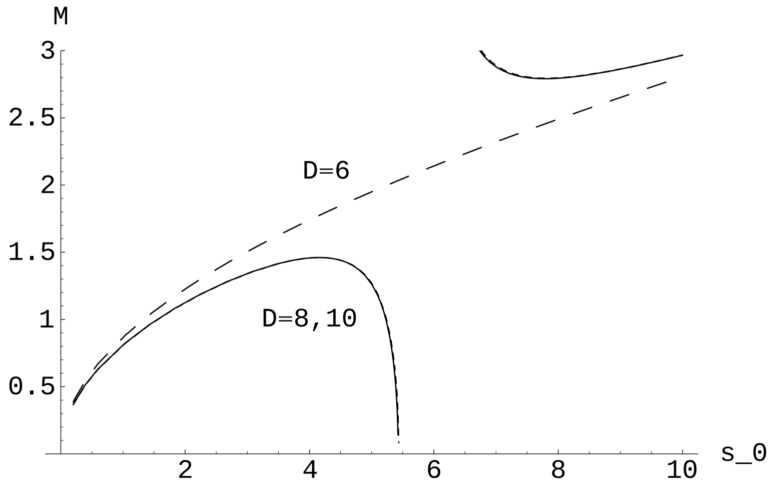


**Figure 7:** The variation of $M$ with $s_0$ corresponding to $\Pi_A$ of the I=2 current (2.3).

interpolating currents, power corrections from condensates with $D \geq 5$ unfortunately turn out to be numerically large for our three interpolating currents. Especially the quark gluon mixed condensate plays a dominant role.

Numerical analysis indicates the stable region of the continuum threshold is around several $\mathrm{GeV}^2$. But OPE of both sum rules converges only when the continuum threshold is very big, $s_0 \geq 10\,\mathrm{GeV}^2$. Such a large value of $s_0$ is irrelevant to the experimentally observed $\Theta^+$ pentaquark. In fact, lack of OPE convergence renders these FESRs very sensitive to the high dimension power corrections.

One may wonder whether the above conclusion is the artifact of FESR approach only. We have also performed the numerical analysis using the Laplace sum rule. Requiring the pole contribution is greater than 40% of the whole sum rule, we arrive at the lower limit of the Borel parameter $M_{\mathrm{min}}$. The convergence of operator product expansion requires the high dimension operators be suppressed. Numerically we may require the ratio between the dimension D condensate and perturbative term is smaller than $1/2^D$. In this way we get the upper limit of the Borel paramter $M_{\mathrm{max}}$. For all the above interpolating currents, we find $M_{\mathrm{min}} > M_{\mathrm{max}}$ for both tensor structures. In other words, there does not exist a working Borel window in the Laplace sum rule analysis. This point has been noted for the spin 1/2 pentaquark case in [27].

Recall both $\phi$ and $\omega$ mesons are narrow resonances above threshold. $\phi$ decays into $K\bar{K}$ and $\omega$ decays into $3\pi$. Their FESRs converge. The extracted vector meson masses agree with the experimental data. The resonance pole contribution dominates the $K\bar{K}$ or $3\pi$ continuum (background). Similarly, if spin 3/2 pentaquarks *really* exists as an extremely narrow resonance as indicated by those positive experiments, its pole contribution should dominate the $KN$ background. Hence one would expect (1) a converging FESR at least for one of the two tensor structures; (2) a strong signal in the working window with the continuum threshold slightly above $m_\Theta^2$. But none of these spin 3/2 pentaquark FESRs satisfies these conditions. This fact strongly indicates the possible nonexistence of spin 3/2



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pentaquarks around 1.53 GeV, which is compatible with the most recent CLAS data [35]. With a ten times larger database, the $nK^+$ spectrum is very smooth around 1.53 GeV. In fact, CLAS found no signal of exotic baryon resonances with $B = +1, S = +1$ up to 2.2 GeV.

Although our present investigation indicates the possible nonexistence of a narrow spin 3/2 pentaquark using the kaon-nucleon color-octet interpolating currents from QCD finite energy sum rule analysis, this is not a *strict* proof yet. An exhaustive study of other interpolating currents in search of excellent OPE convergence is very desirable. Only after OPE convergence is established, may one be able to judge the existence of pentaquarks rigorously from the behavior of QCD sum rules. One important scheme is to include the coupled channel effects by introducing a mixed interpolating current which is a combination of several interpolating currents with different color structures. The mixing and coupled channel effect may help suppress the high dimension condensates and stabilize the sum rule. Work along this direction is in progress.

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